

Precise Plasmonic Spectra of Gold and Silver Clusters Using the DFT+U Method

Hans-Christian Weissker

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European Theoretical Spectroscopy Facility, www.etsf.eu



Note: animations disabled

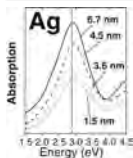
Outline

- 1 Introduction
- 2 Method: TDDFT / Time Evolution
- 3 Surface Plasmons / Electron-Density Oscillations
 - Bare Clusters
 - Nanorods
 - Approximations in (TD)DFT Calculations
- 4 DFT+U for Noble-Metal Clusters
- 5 Conclusions

Plan

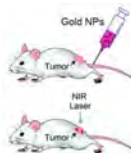
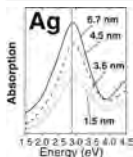
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Surface Plasmons in Metal Clusters



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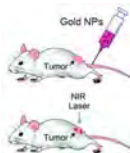
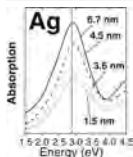
Applications



- Manyfold applications
 - Biomolecule labeling and sensing
 - Cancer therapy / Medical imaging
 - Surface-enhanced Raman Scattering
 - Nanoplasmonics
 - Antibacterial effects
 - Photo-catalysis
 - Solar cells, ... etc. etc.

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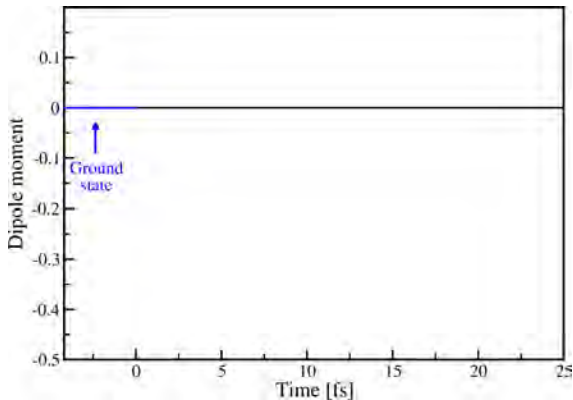
**Theoretical
understanding?
Tune plasmon energy
Materials ??? (Cost!)**

Help understand experiment / engineer desired prop't's!

Plan

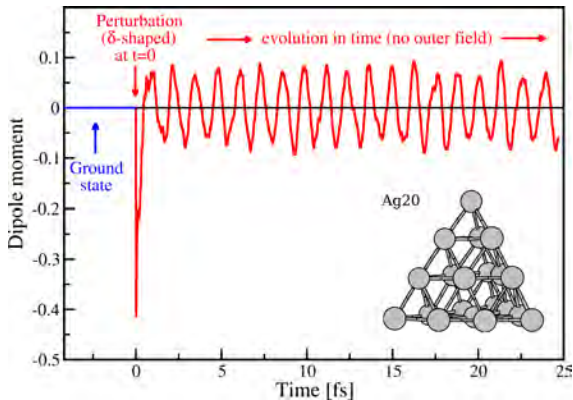
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The Method: Time-Evolution Formalism of TD-DFT



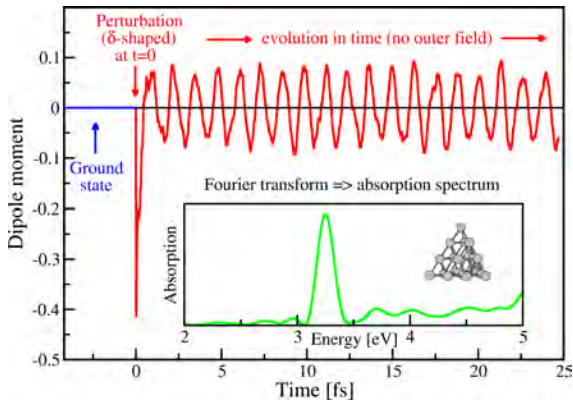
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- Norm-cons. pseudo-pot.
- For optics: PBE, PBE+U, ...

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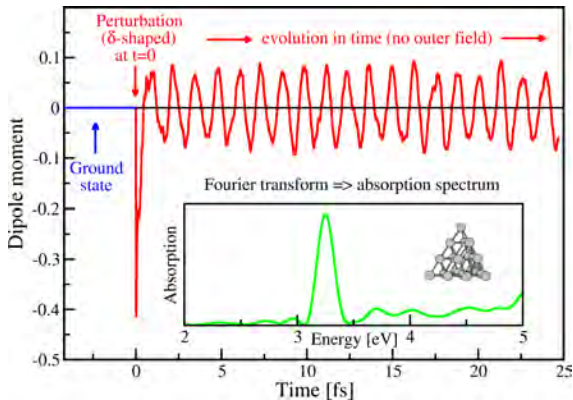
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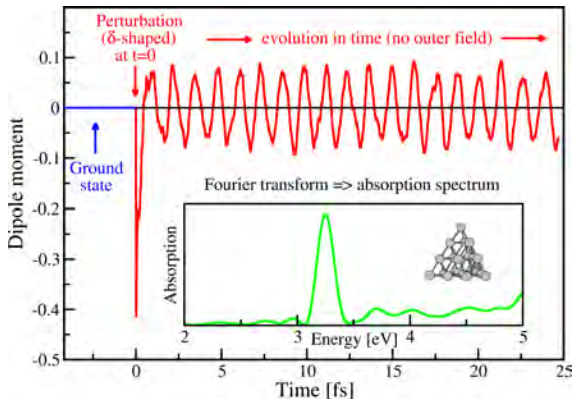
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✓ Advantage: no large number of unoccupied states needed

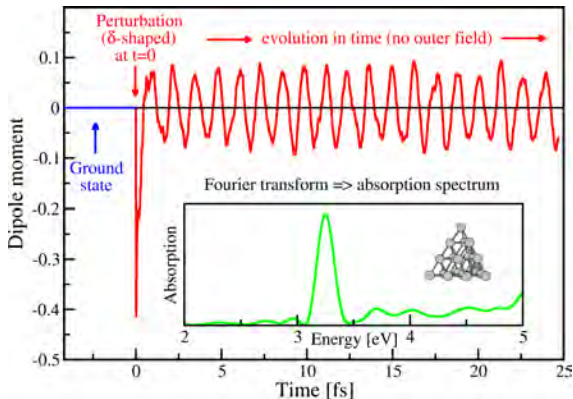
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- ✓ Disadvantage: no direct information about transitions

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- For optics: PBE, PBE+U, ...

- ✓ Advantage: no large number of unoccupied states needed
- ✓ Disadvantage: no direct information about transitions
- ✓ **Advantage: explicit view of charge dynamics**

Yabana & Bertsch, PRB **54**, 4484 (1996).

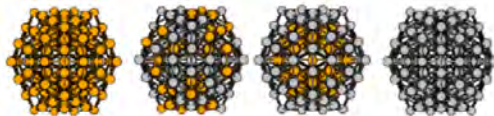
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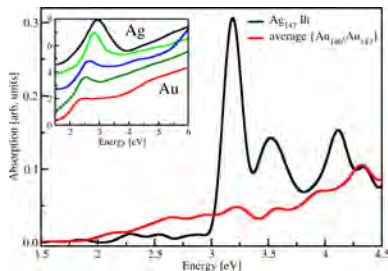
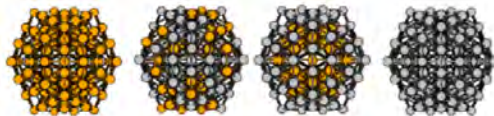
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Pure and Alloyed Bare Clusters – General Results

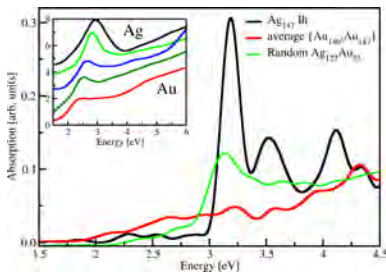
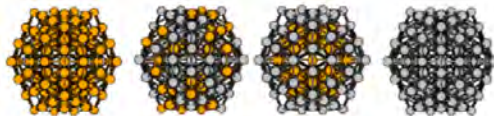


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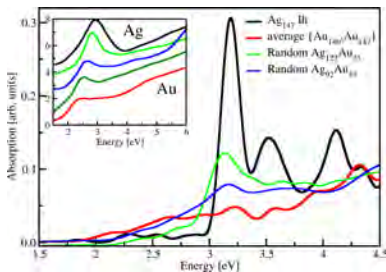
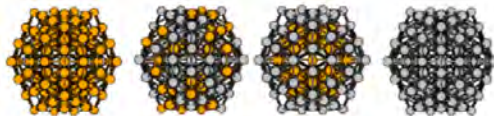
López-Lozano & Weissker, JPCC 117, 3062 (2013).

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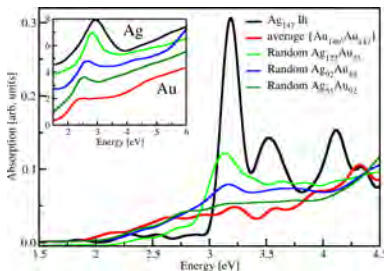
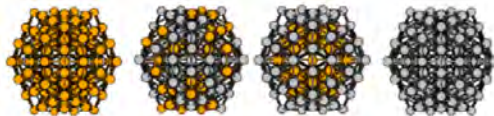
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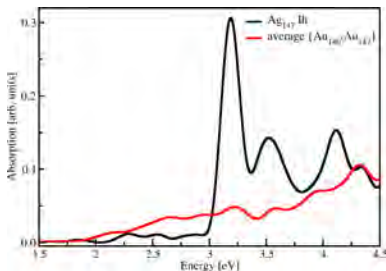
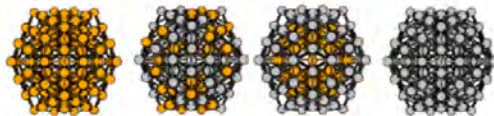
✓ Agreement experiment-theory

Redshift? Size distribution in experiment? Matrix?

Temperature? Spin-orbit coupling? Ligands? v_{XC} & f_{XC} ...

✓ Ag vs. Au & alloying

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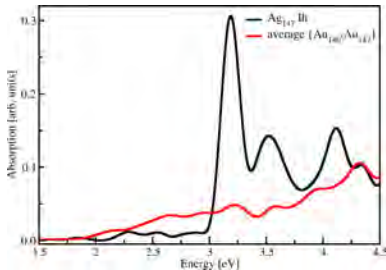
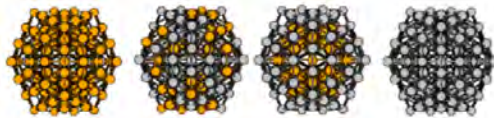
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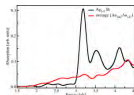
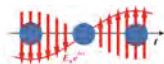
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⇒ **LSPR as charge oscillation?**

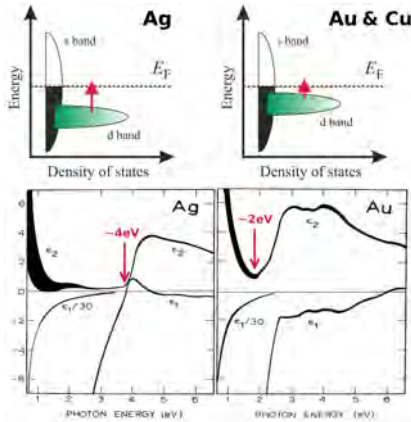
Icosahedra Ag₁₄₇ and Au₁₄₇

Weissker & López-Lozano, PCCP **17**, 28379 (2015).

- Density oscillation $\leftrightarrow$ classical picture
- $\rho(t, \mathbf{r}) - \rho_{\text{GS}}(\mathbf{r})$; Ag vs. Au



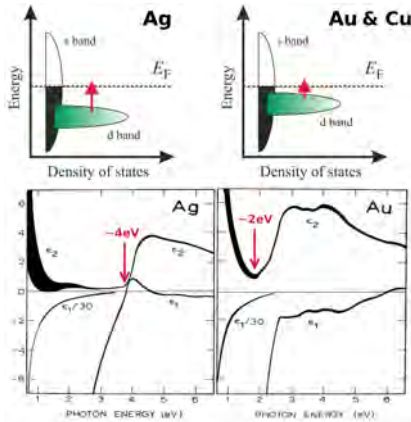
Interband Transitions: Silver vs. Gold



- “Interband” transitions from d band
- Interband onset:
 - Ag at ~4 eV
 - Au & Cu at ~2 eV

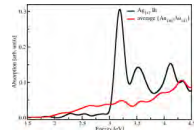
Johnson & Christy, PRB, 1972

Interband Transitions: Silver vs. Gold



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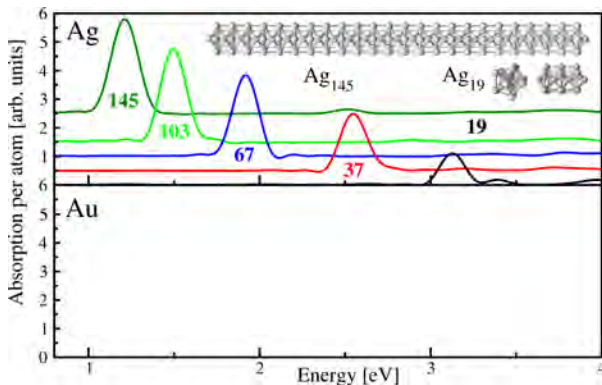
- “Interband” transitions from d band
- Interband onset:
 - Ag at ~4 eV
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- Size just below ≈ 2 nm:
 - Ag shows LSPR,
 - Au & Cu do not.



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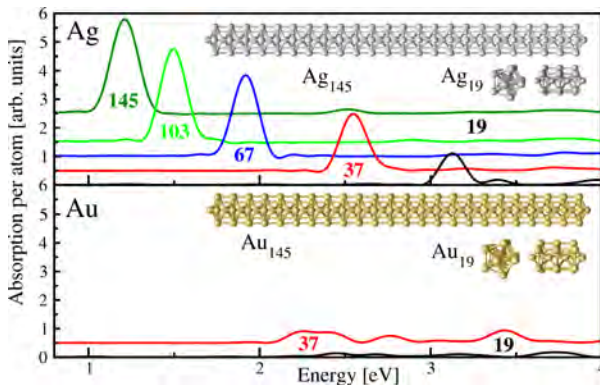
Nanorods: Aspect-Ratio-Dependent Emergence of SPR in Gold



- Aspect-ratio-dependent longitudinal resonance: Mie or TDDFT
- Short Au rods: no strong resonance

López-Lozano, Barron, Mottet & Weissker, PCCP **16**, 1820 (2014).

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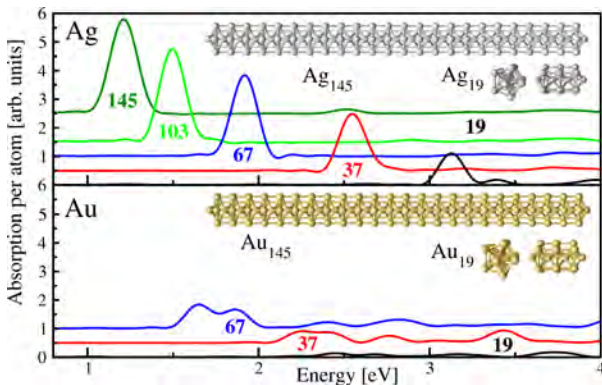


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Nanorods Ag_{37} and Au_{37}

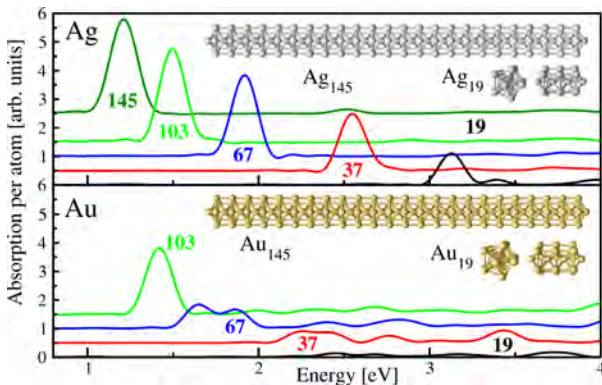
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(M. Stener; Ch. Aikens)

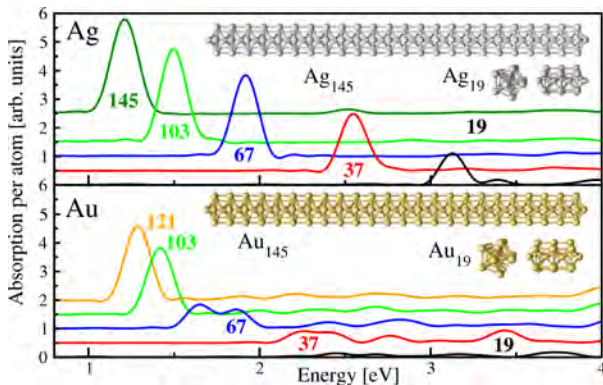
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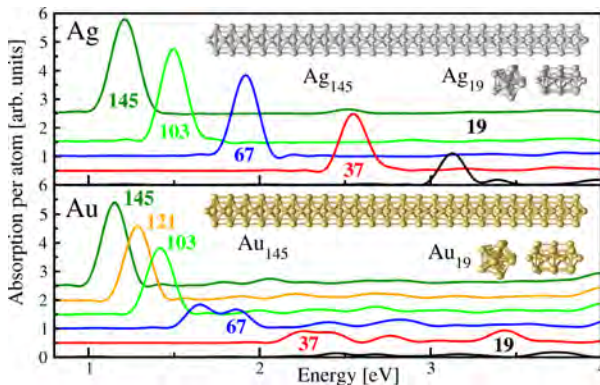
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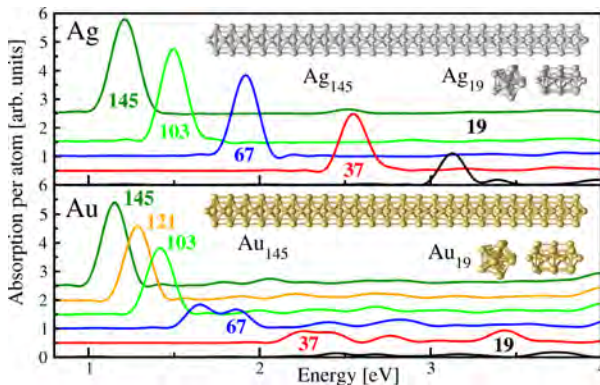
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- High aspect-ratio $\Rightarrow$ strong resonance; gold becomes silver-like

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- High aspect-ratio $\Rightarrow$ strong resonance; gold becomes silver-like
- Decoupling from interband transitions from d electrons

López-Lozano, Barron, Mottet & Weissker, PCCP **16**, 1820 (2014).

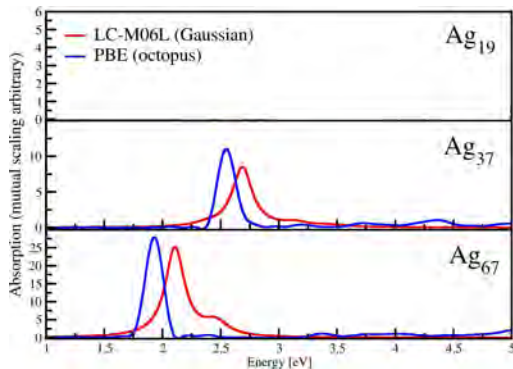
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Approximations for exchange and correlation (v_{xc} , f_{xc})

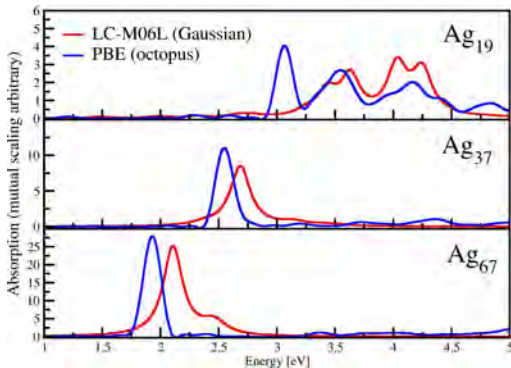
- Absorption in pentagonal silver rods, excitation || axis



Rajarshi Sinha-Roy, [...], H.-Ch. Weissker, F. Rabilloud, A.I. Fernández-Domínguez, ACS Phot., **4**, 1484 (2017)

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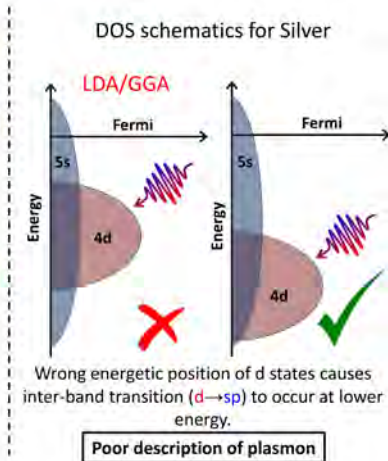
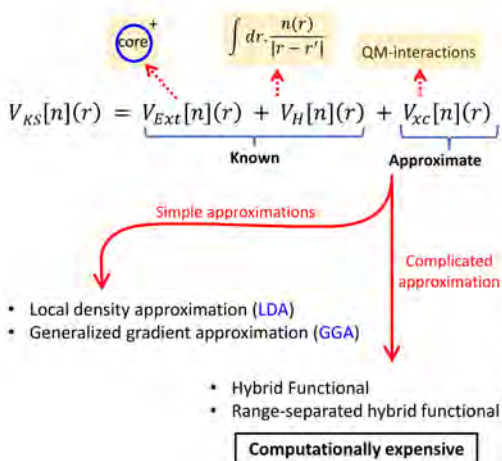
- LDA / GGA fine for LSPR well decoupled from d-electron excitations; fails badly if strong coupling with d excitations

Rajarshi Sinha-Roy, [...], H.-Ch. Weissker, F. Rabilloud, A.I. Fernández-Domínguez, ACS Phot., **4**, 1484 (2017)

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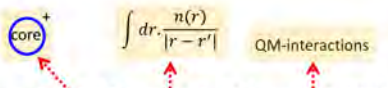
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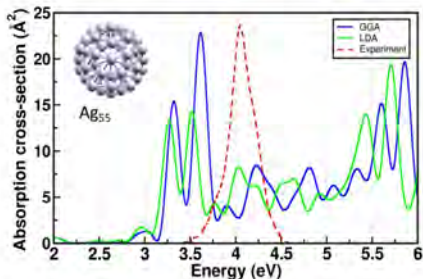
The d band in noble-metals with DFT and DFT+U



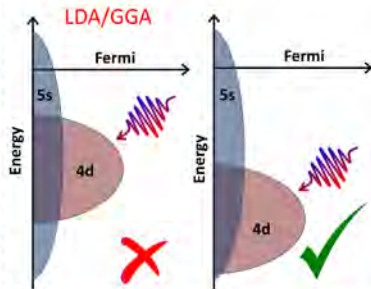
The d band in noble-metals with DFT and DFT+U

$$V_{KS}[n](r) = V_{Ext}[n](r) + V_H[n](r) + V_{xc}[n](r)$$





DOS schematics for Silver



Wrong energetic position of d states causes inter-band transition ($d \rightarrow sp$) to occur at lower energy.

Poor description of plasmon

The d band in noble-metals with DFT and DFT+U

$$E_{DFT+U}(n, \{n_{mm'}^{l,\sigma}\}) = E_{DFT}(n) + \underbrace{E_{ee}(\{n_{mm'}^{l,\sigma}\}) - E_{dc}(\{n_{mm'}^{l,\sigma}\})}_{\text{Correction term } (E_u)}$$

l : atomic site index

σ : spin

m, m' : angular quantum number

$n_{mm'}^{l,\sigma}$: density matrix in localized atomic basis

$$E_u(\{n_{mm'}^{l,\sigma}\}) = \sum_{l,n,l} \frac{U_{l,n,l}^{eff}}{2} \sum_{m,\sigma} \left[n_{mm}^{l,n,l,\sigma} - \sum_{m'} n_{mm'}^{l,n,l,\sigma} n_{mm'}^{l,n,l,\sigma} \right]$$

dropping n and l index

$$E_u(\{n_{mm'}^{l,\sigma}\}) = \sum_l \frac{U_l^{eff}}{2} \sum_{m,\sigma} \left[n_{mm}^{l,\sigma} - \sum_{m'} n_{mm'}^{l,\sigma} n_{mm'}^{l,\sigma} \right]$$

①

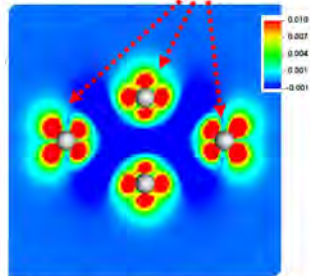
Increases
Energy

②

Decreases
Energy

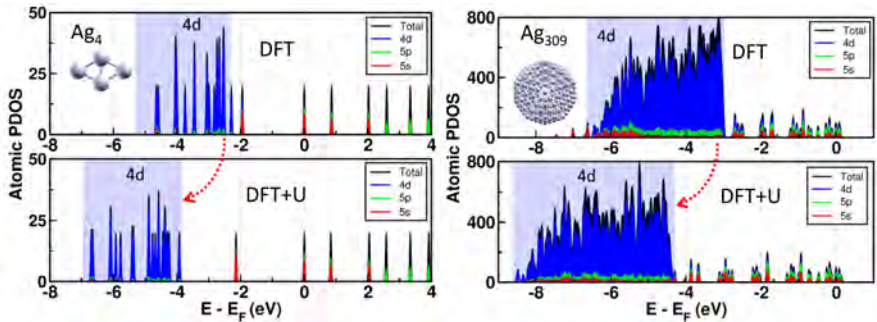
Correction term (E_u)

Electron
accumulation after U
correction



Total charge density difference between DFT+U (4eV) and DFT calculation with GGA

PDOS before and after adding the U correction

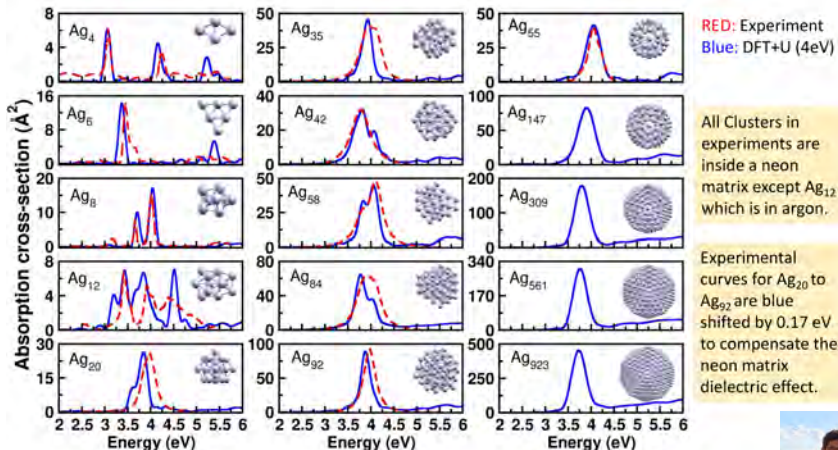


- Use $U = 4$ eV, shown to give good results for bulk silver

Avakyan *et al.*, Optical Materials, **109**, 110264, (2020)

- shifts d edge from ≈ 3 eV to ≈ 4 eV after applying the correction; consistent with photoemission spectra in the literature

Spectra of silver clusters using DFT+U



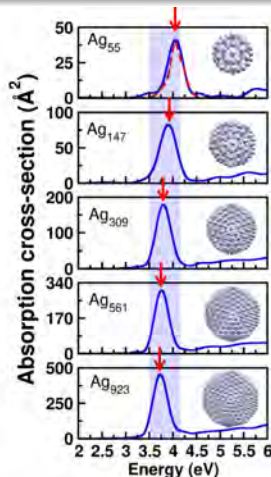
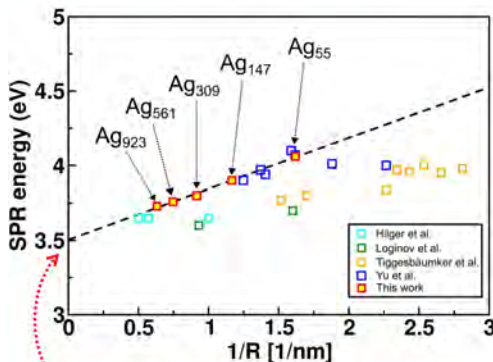
- DFT+U yields excellent spectra very efficiently (size!)
- Transferability: one U value works for all sizes



Mohit Chaudhary

SPR energies of silver clusters using DFT+U

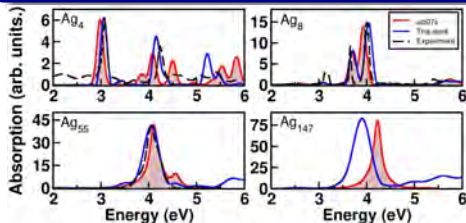
Note 12/2024: For updated results, please see this published article



- SPR energies in agreement with experiment. Size dependence!
- Transferability: one U value works for all sizes **to be submitted**

Now published: Nat. Commun. 15, 9225 (2024).

Comparison with previous calculations



Comparison with RS-hybrid functional:

- We obtained better agreement with experiments in our calculations
- Calculation with RS-hybrid limited to Ag₁₄₇

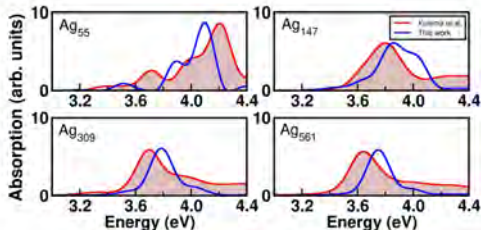
¹Schira & Rabilloud, J. Phys. Chem. C, 123(10), 2019

²Rabilloud, J. Phys. Chem. A, 117(20), 2013

Comparison with mGGA functional

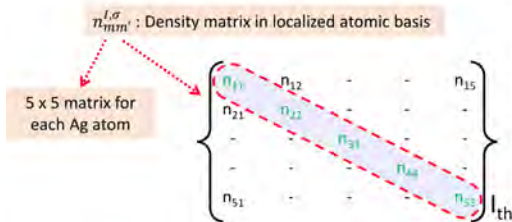
- For smaller cluster (Ag₅₅), overestimation of LSPR.
- For bigger clusters, underestimation of LSPR

³Kuisma et al., PRB, 91(11), 2015



- Range-separated hybrid functions good but *heavy*
- A meta-GGA functional: good for large clusters...

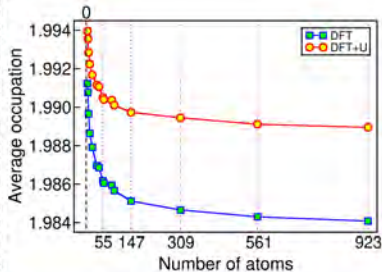
Size dependent localization of 4d electrons



$$\text{Average Occupation} = \frac{1}{\text{atoms}} \sum_l \frac{\text{Trace}\{n_{mm'}^{l,\sigma}\}}{5}$$

Higher
occupation of
d-orbitals

Lesser
screening of s
electrons

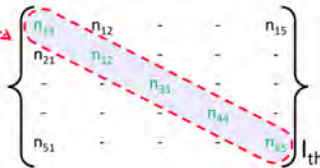


- With U correction, average d electron localization increases
- With increasing size, average occupation decreases, or screening because of d-electrons increases.
- Higher screening $\rightarrow$ lower value of LSPR

Size dependent localization of 4d electrons

$n_{mm'}^{l,\sigma}$: Density matrix in localized atomic basis

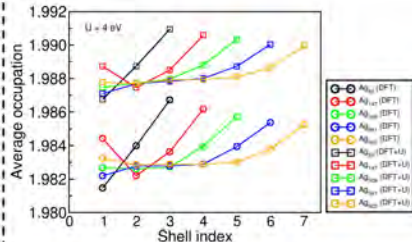
5 x 5 matrix for each Ag atom



$$\text{Average Occupation} = \frac{1}{\text{atoms}} \sum_l \frac{\text{Trace}\{n_{mm'}^{l,\sigma}\}}{5}$$

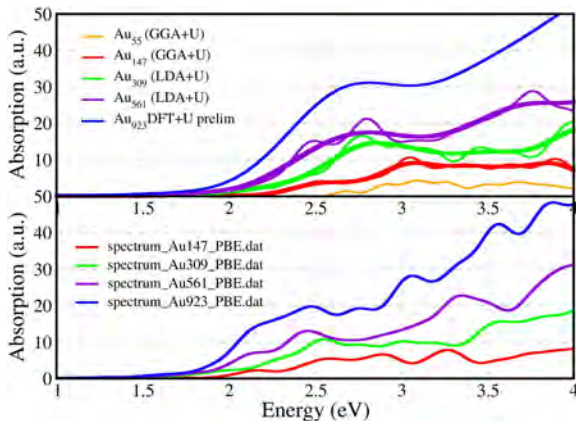
Higher
occupation of
d-orbitals

Lesser
screening of s
electrons



- With U correction, average d electron localization increases.
- Outermost two shells have higher average localization, pointing towards reduced screening of s-electrons on the surface

DFT+U for Gold Clusters



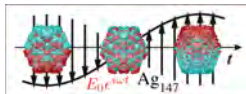
- Very preliminary: DFT+U improves description of SPR emergence
- Small clusters: no good agreement

Plan

- 1 Introduction
- 2 Method: TDDFT / Time Evolution
- 3 Surface Plasmons / Electron-Density Oscillations
 - Bare Clusters
 - Nanorods
 - Approximations in (TD)DFT Calculations
- 4 DFT+U for Noble-Metal Clusters
- 5 Conclusions

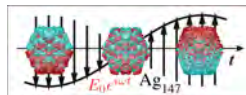
Conclusions

- TDDFT for “Quantum-sized” Ag and Au clusters: LSPR as charge oscillation
 - Difference Ag vs. Au; d screening
 - Aspect-ratio-dependent red shift may decouple LSPR from interband transitions in rods



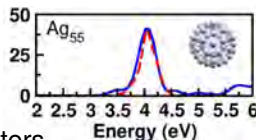
Conclusions

- TDDFT for “Quantum-sized” Ag and Au clusters:
LSPR as charge oscillation
 - Difference Ag vs. Au; d screening
 - Aspect-ratio-dependent red shift may decouple LSPR from interband transitions in rods



- **Excellent silver spectra using DFT+U**

- Excellent agreement for **small and large** clusters
 - Size dependence of SPR energies: agreement with experiment
 - Using the **same value of U for all sizes up to 1000 atoms (3 nm)**
- ⇒ Opens the possibility for calculations coupling the clusters to molecules, among each other, etc. — in other words, to describe many of the situations found in experiments and applications.



~~to be submitted~~

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