Packing pentagons: from Albrecht Dürer to molecular detection

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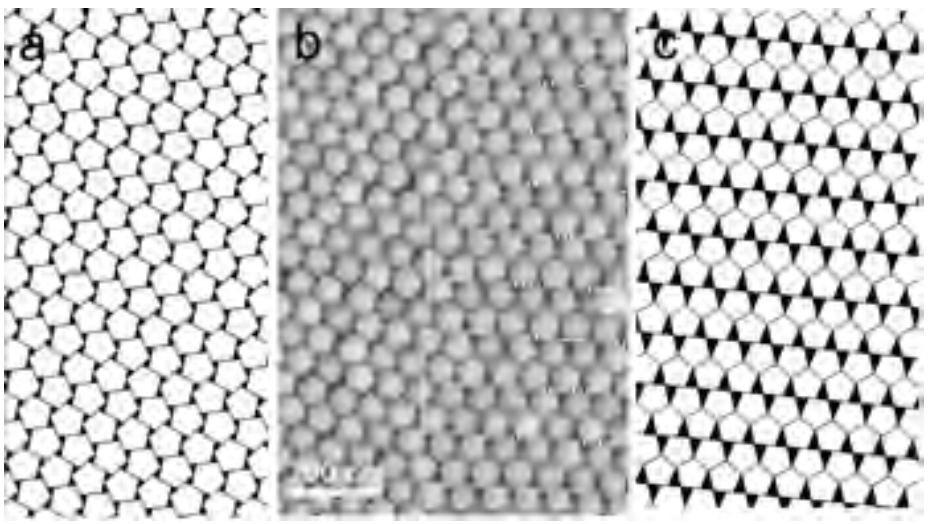
25 October 2023

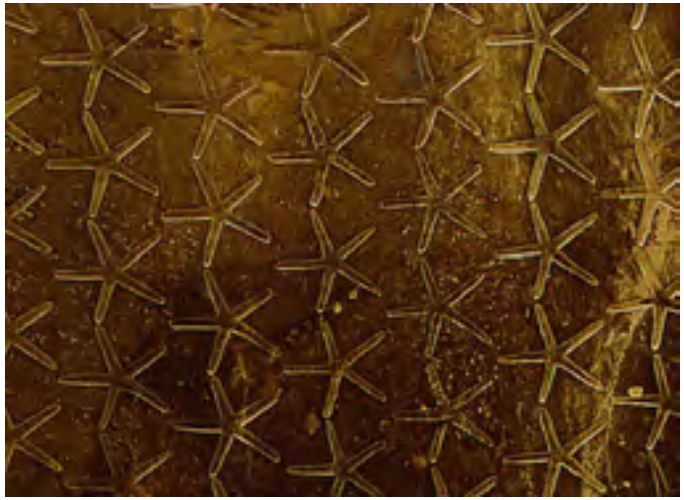


Our team

- Laboratoire de Physique des Solides, Orsay
  - Jules Marcone
  - Wajdi Chaâbani
  - Claire Goldmann
  - Marianne Impéror
  - Cyrille Hamon

(Pre)-history I

Ancient Egypt : 2300 BC (?)



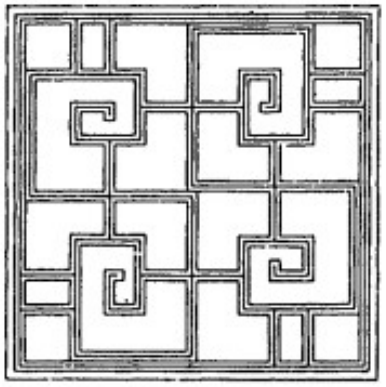
Z. Feisal, dx.doi.org/10.21608/erjm.2012.67177

T. Hales and W. Kusner,

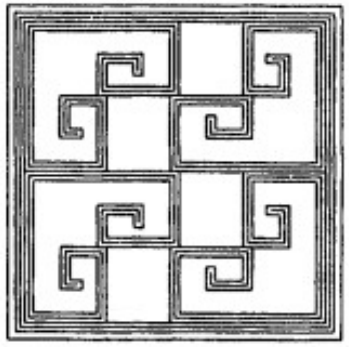
jiggerwit.wordpress.com/2016/09/16/pentagon-packings/

(Pre)-history II

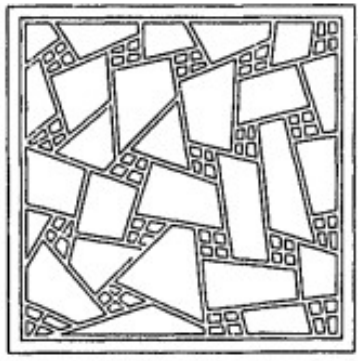
Ancient China : 1000 BC (?) – 1900 AD



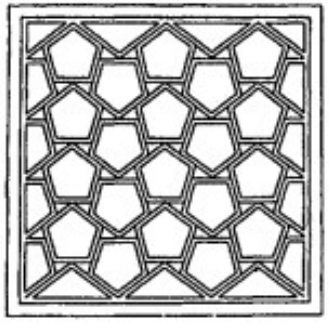
V: S-SCROLL, 286



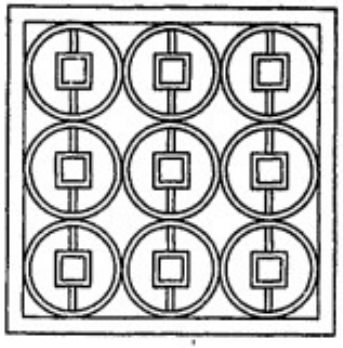
W: U-SCROLL, 292



X: RUSTIC ICE-RAY, 298



Y: SYMMETRY ICE-RAY, 306



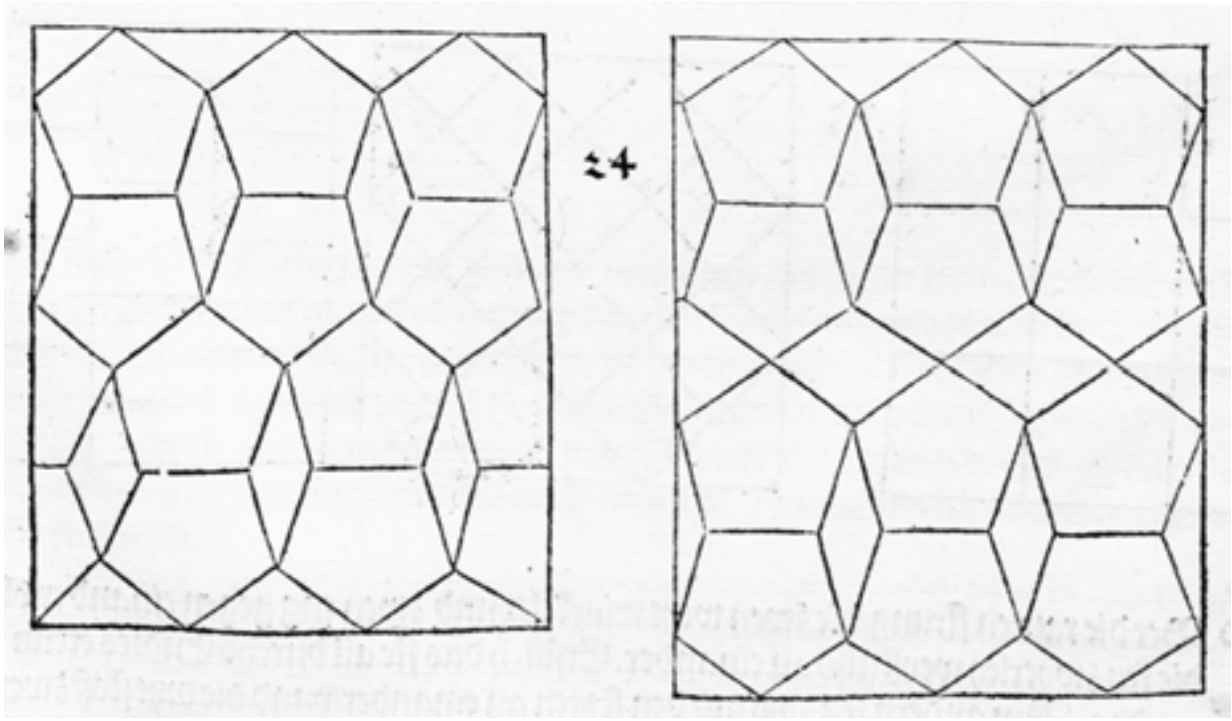
Z: SQUARE-ROUND, 318

&: SUPPLEMENT, 330

D. S. Dye, *Chinese Lattice Designs*, 2nd ed., Dover Publications (2003).

(Pre)-history III

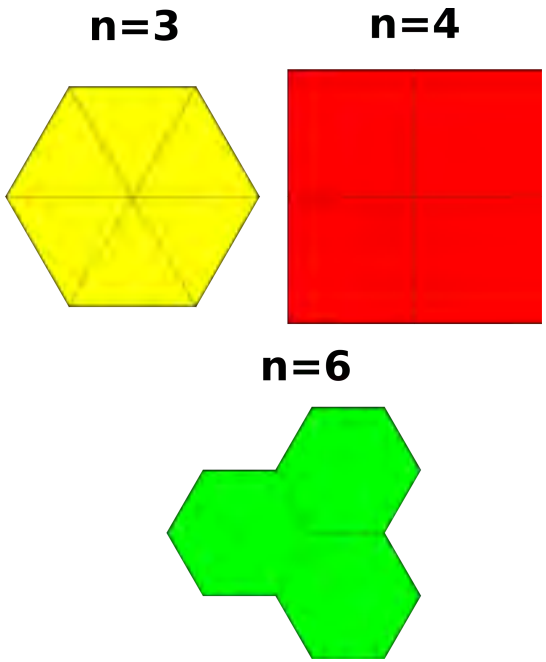
The Renaissance
Albrecht Dürer (1525)



A Dürer, *Unterweysung der Messung...*, Nüremberg (1525).

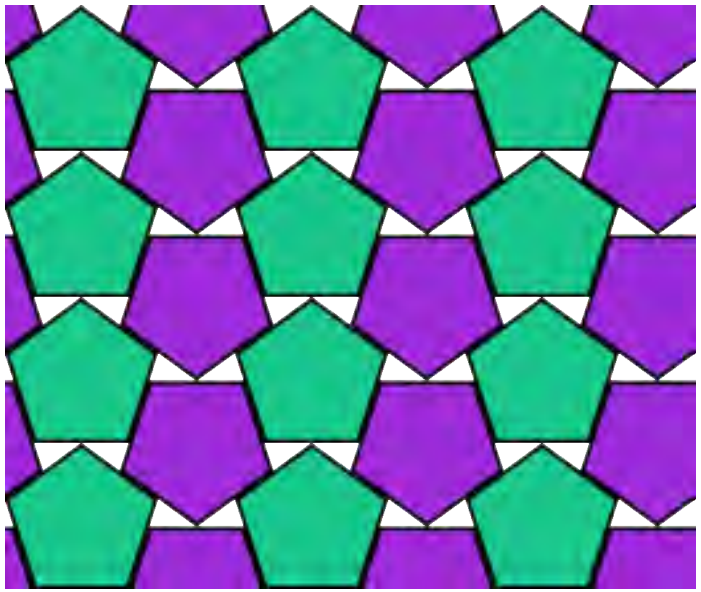
Tiling the plane

Regular polygons



$\eta = 1$

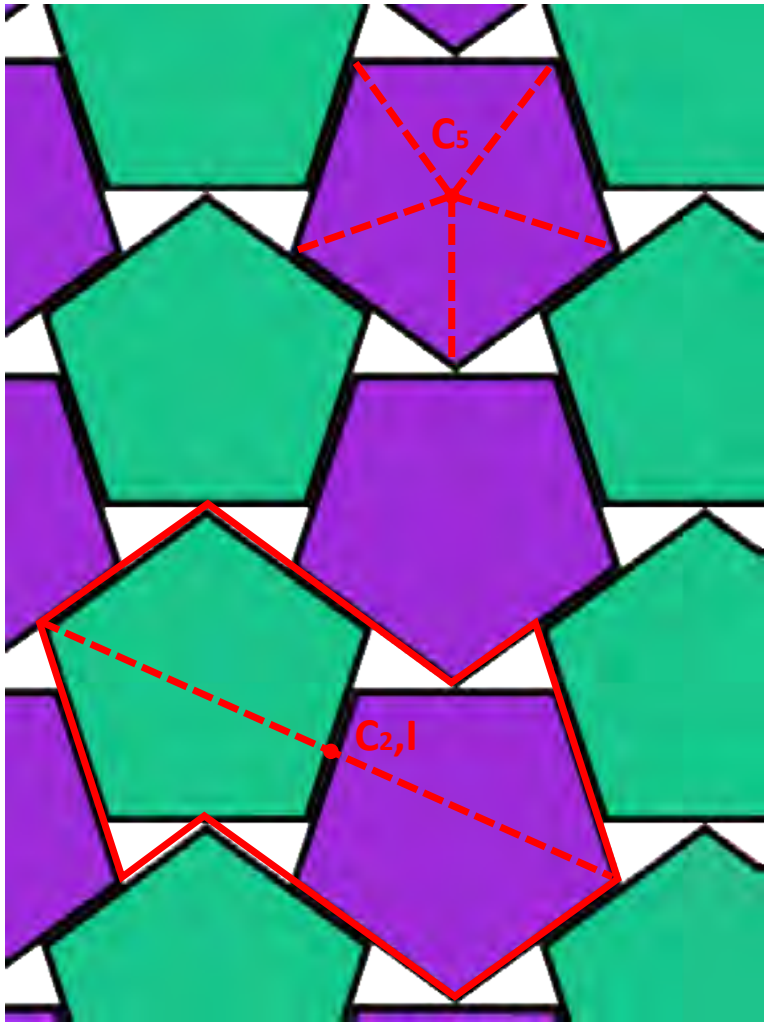
Pentagons ?



Ice ray : $\eta = 0.92131067\ldots$

Hales and Kusner, arXiv :1602.07220 (2016).
https://softqc.wordpress.com/packing-of-hard-regular-pentagons-pentagonal-ice-ray/
© Tom Ruen, © Toby Hudson

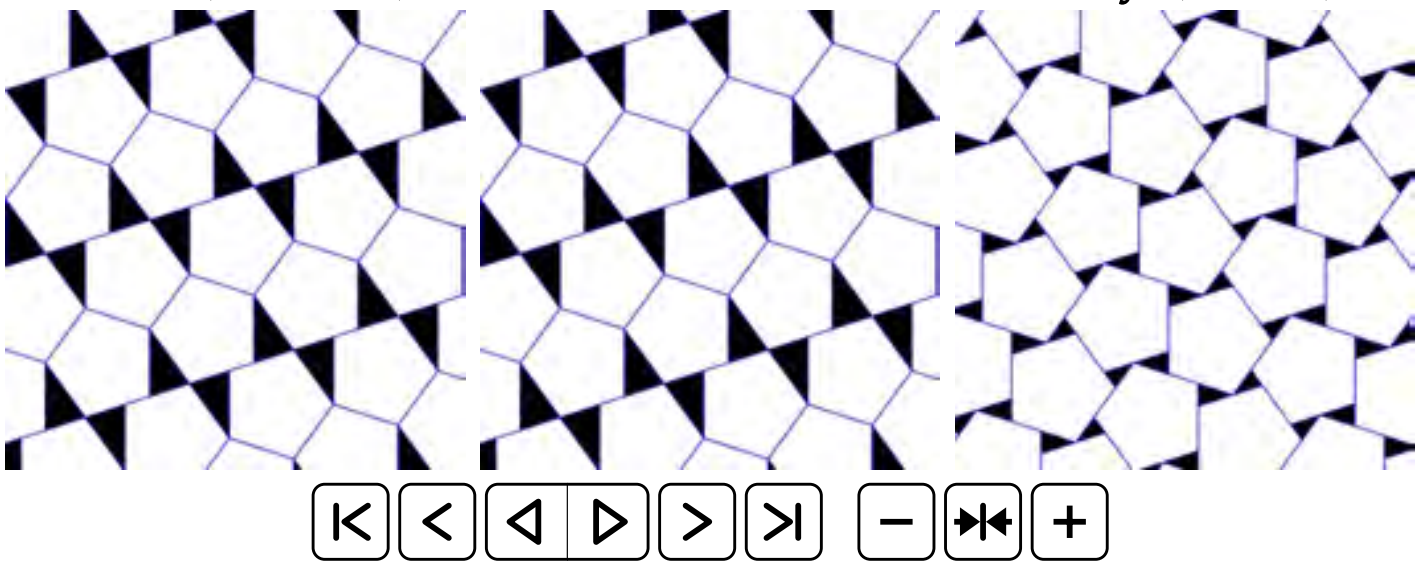
Symmetry



Dürer to ice-ray (and back)

Continuous one-parameter family of double lattices :

Dürer ($u = 0.5$) Ice-ray ($u = 0$)

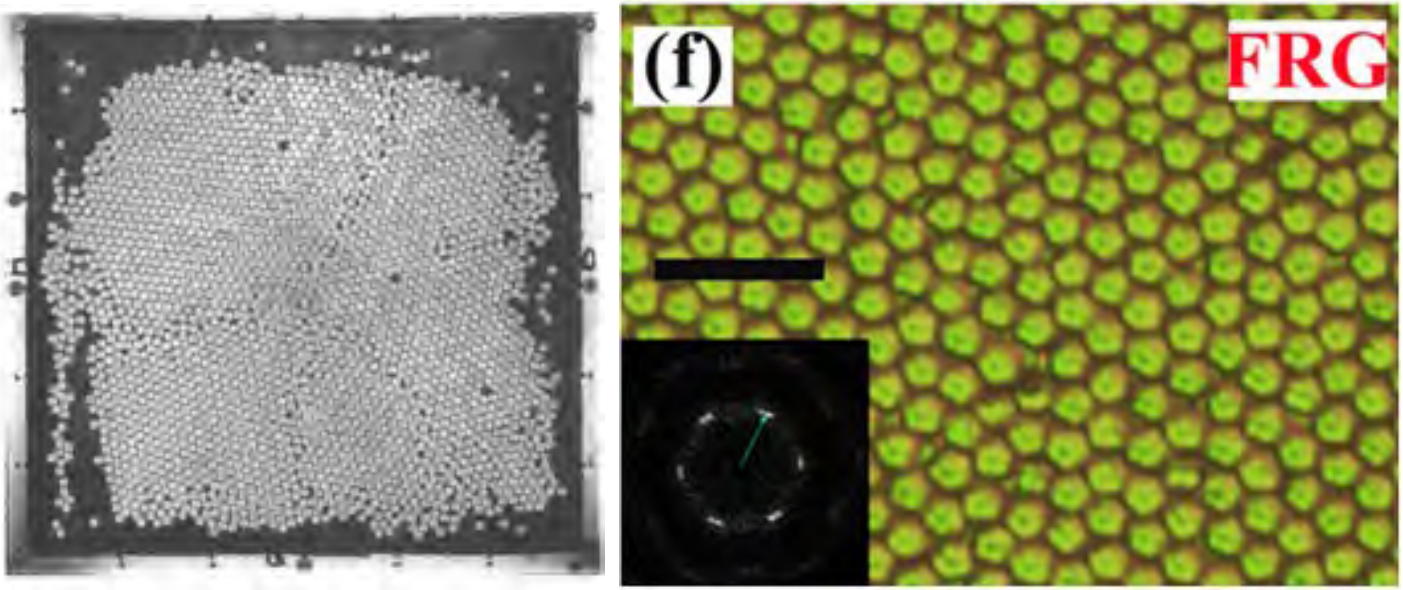


G. Kuperberg and W. Kuperberg, *Discrete Comput. Geom.* **5**, 389 (1990).

Experimental results I

Centimeters

Micrometers

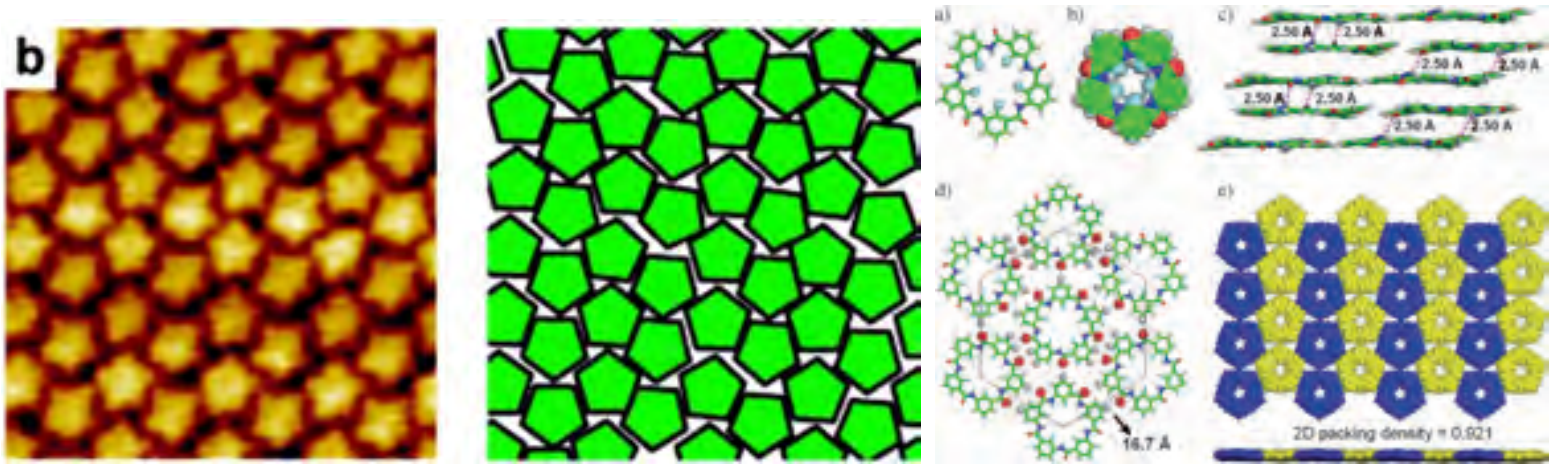


First evidence of the ice-ray structure.

Y. L. Duparcmeur et al., *J. Phys. : Condens. Matter* **7** 3421 (1995).
Zhao and Mason, *Phys. Rev. Lett.* **103** 208302 (2009).

Experimental results II

Ångströms



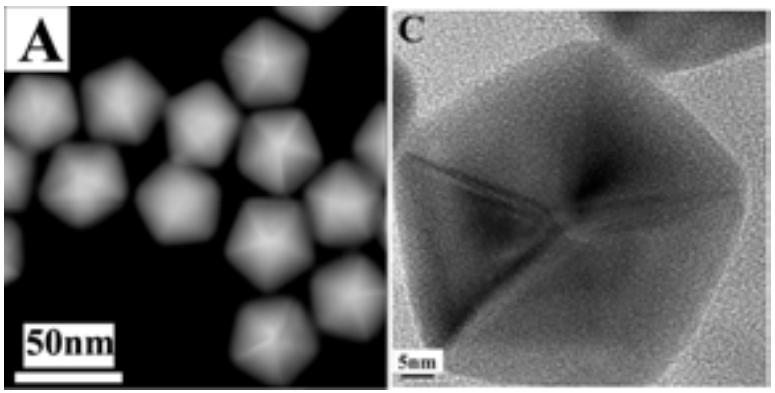
Molecular pentamers on surfaces or in layered stacks

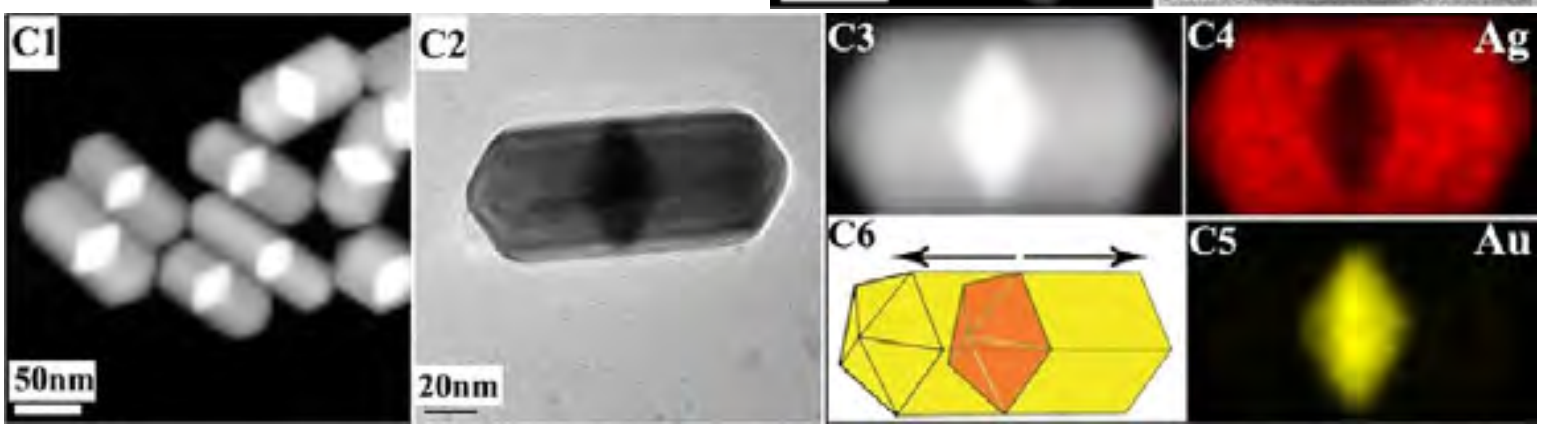
T. Bauert et al., *J. Amer. Chem. Soc.* **131** 3460 (2009).
C. Ren et al., *Angew. Chem.* **50** 10612 (2011).

Building blocks I

Silver nanoprisms

- Pentatwinned Au decahedra as seeds
- Controlled Ag overgrowth
- Kinetics tuned by T and ammonium hydroxide

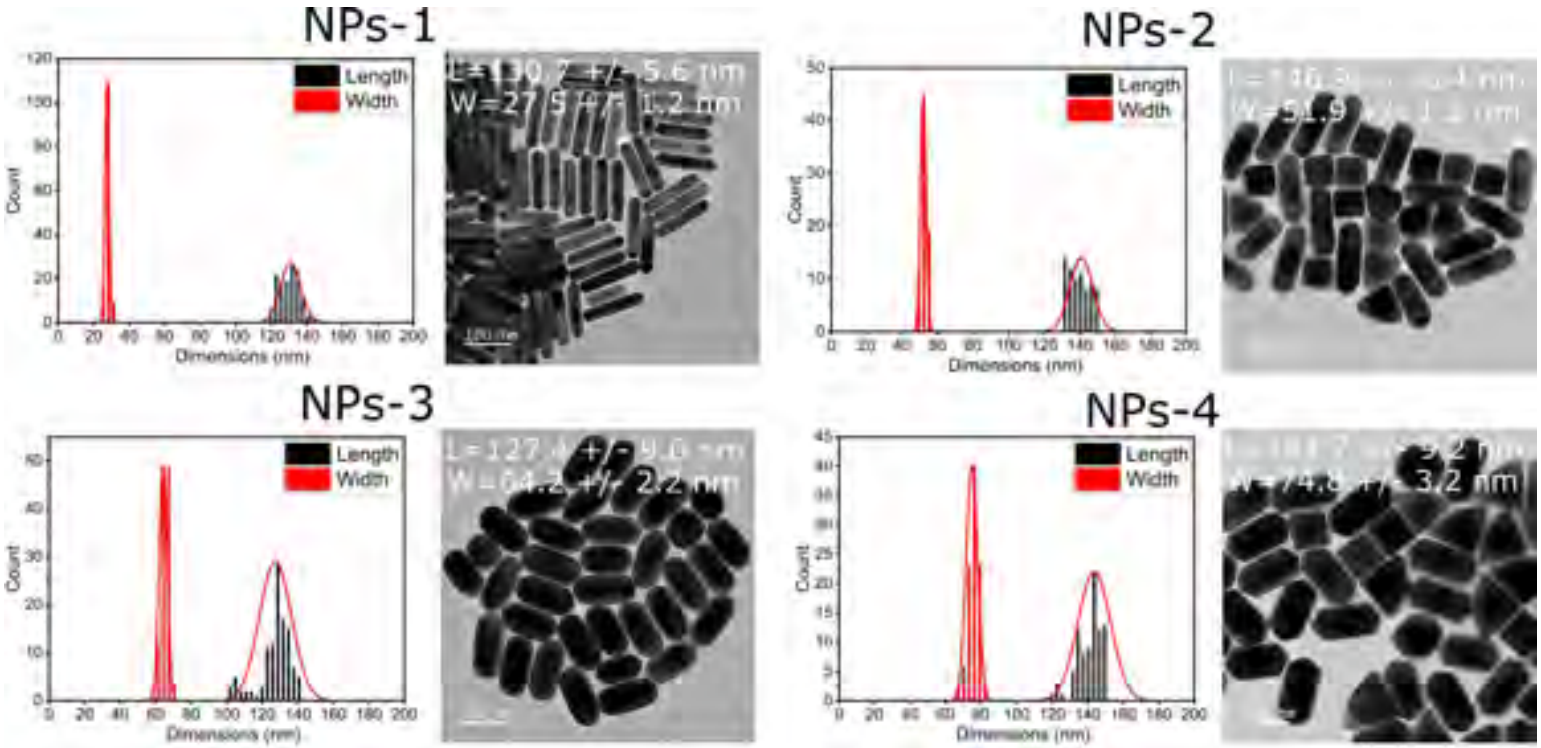




Y. Yang et al., *Chem. Mater.* **25**, 34 (2013).

Building blocks II

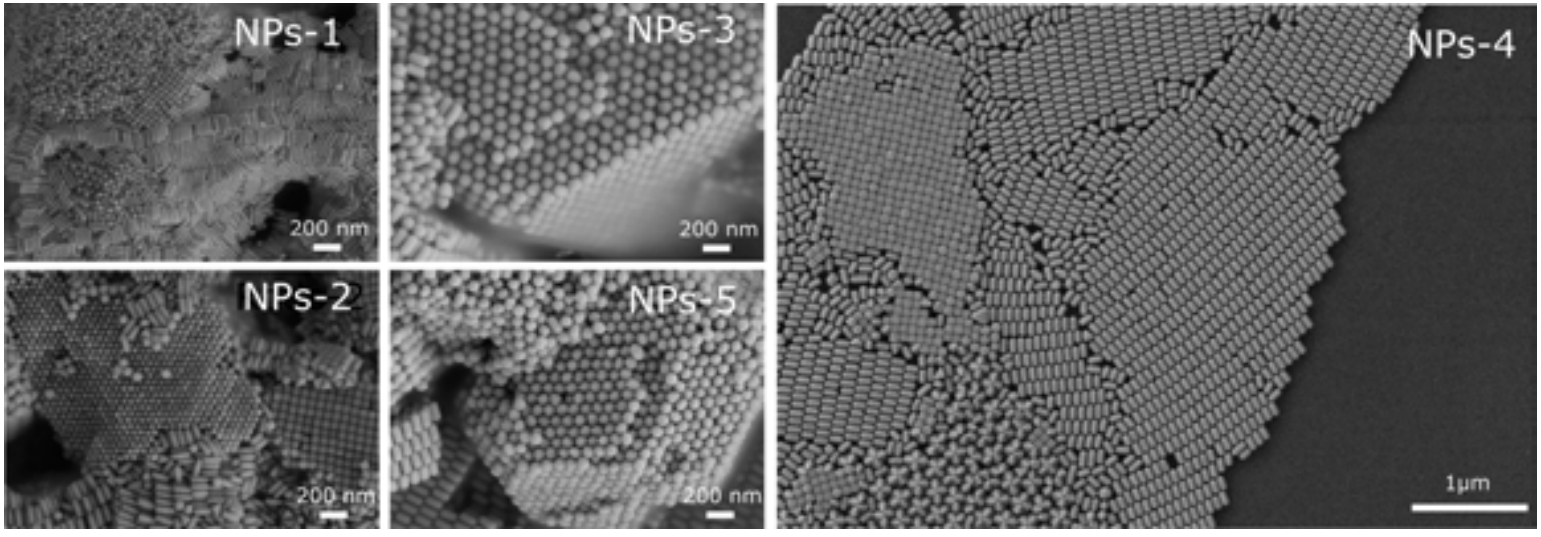
Length : 140 nm, width 30–80 nm



Good size monodispersity, but many isometric by-products

Evaporation-induced self-assembly I

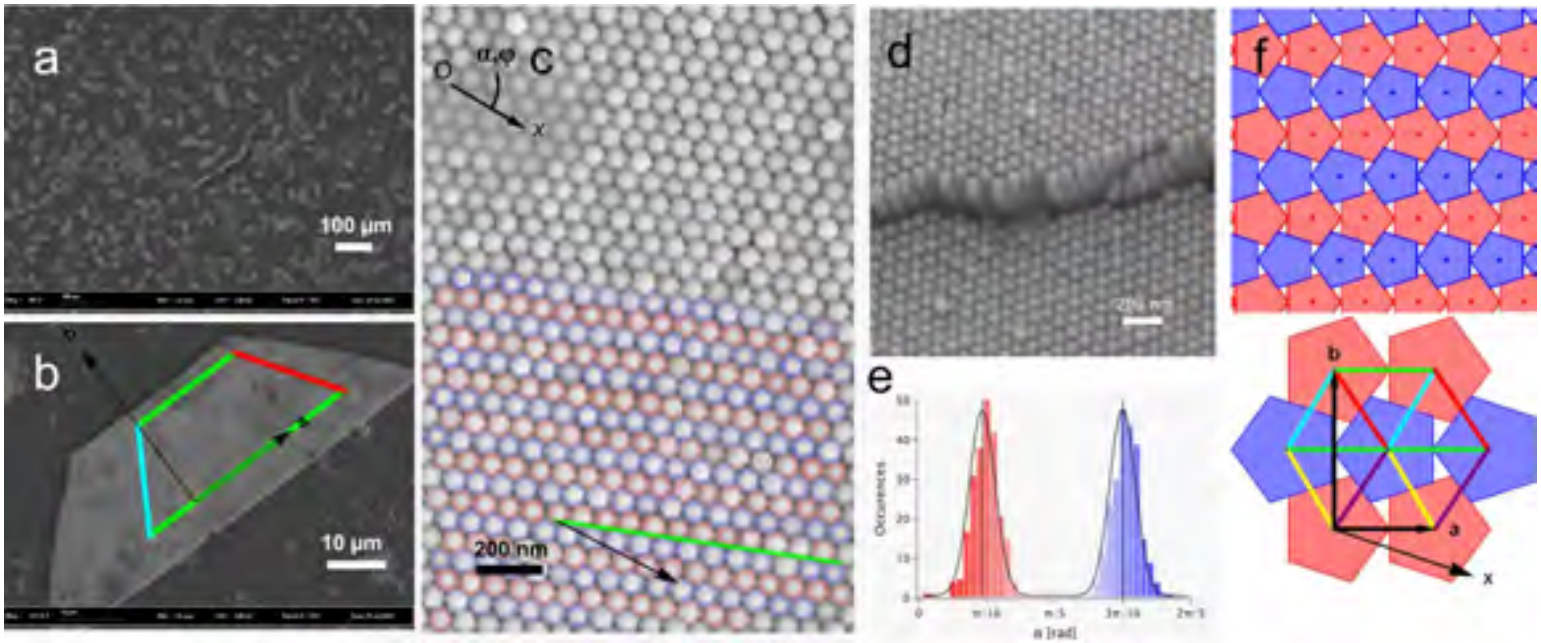
Before shape purification



Disoriented conglomerates of prisms, cubes and tetrahedra, with imperfect shape sorting

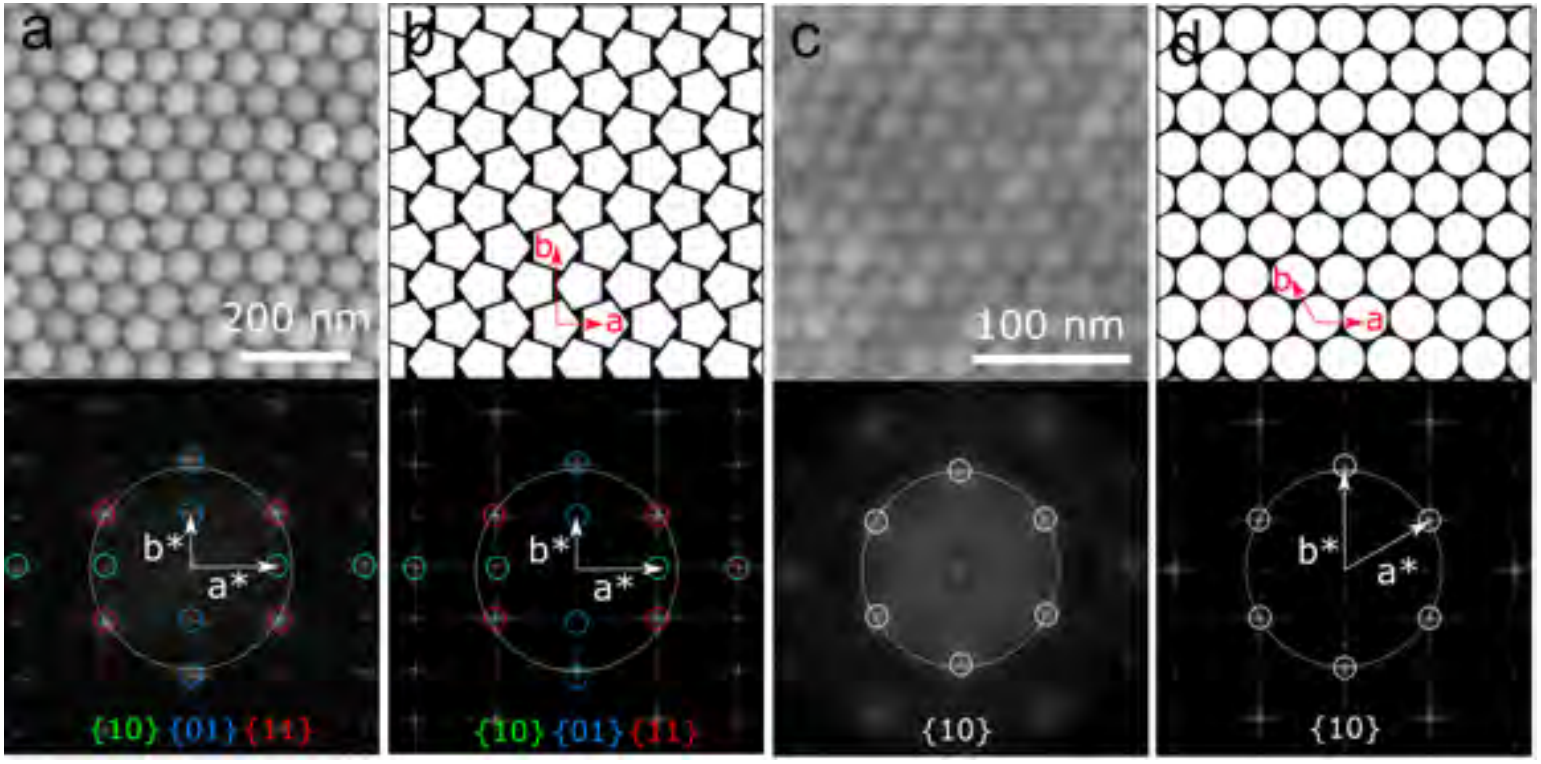
Evaporation-induced self-assembly II

After shape purification



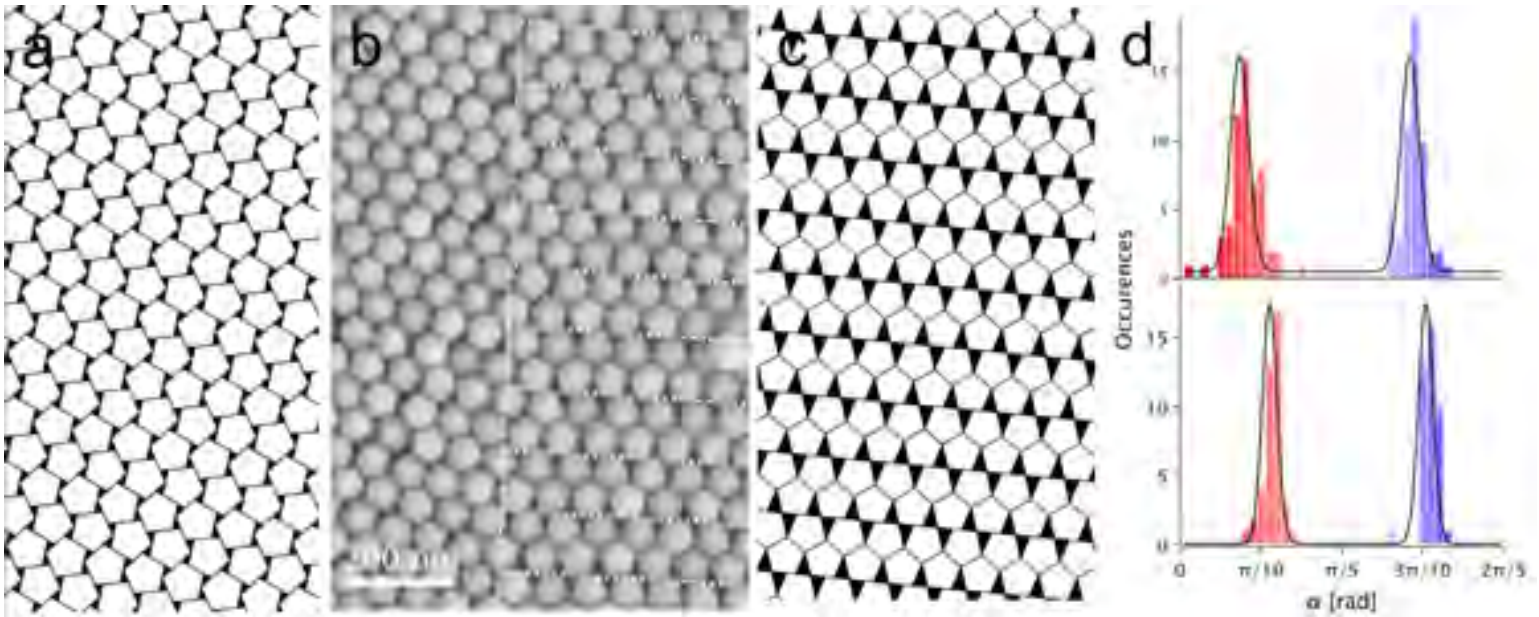
- Supercrystals of a specific (trapezoidal) shape
- consisting of layers of standing prisms
- with orientational order

Lattice symmetry



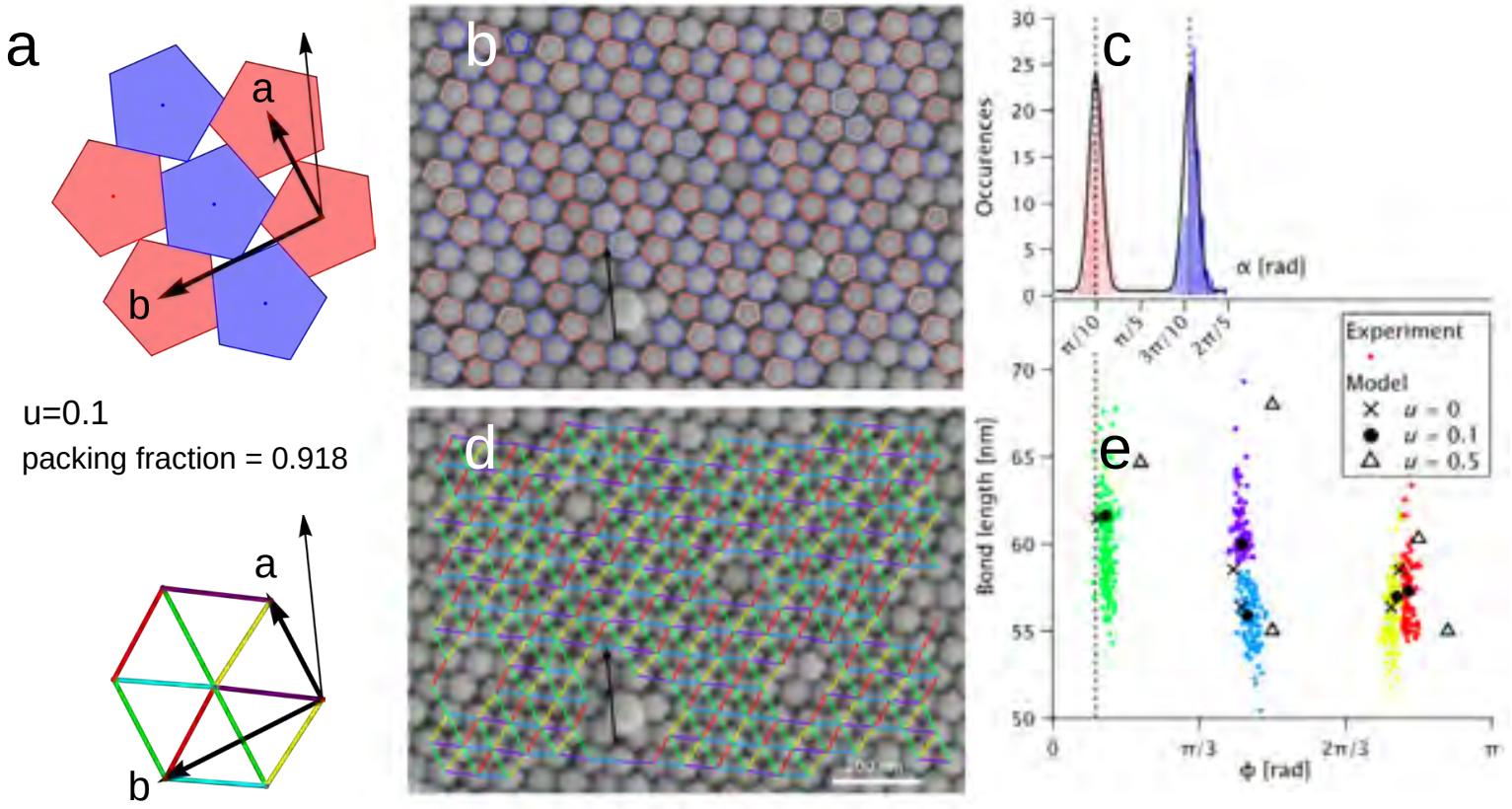
- Rectangular lattice with two particles per unit cell
- and with long-range positional order
- unlike the hexagonal lattice of monocrystalline nanorods!

Structure coexistence



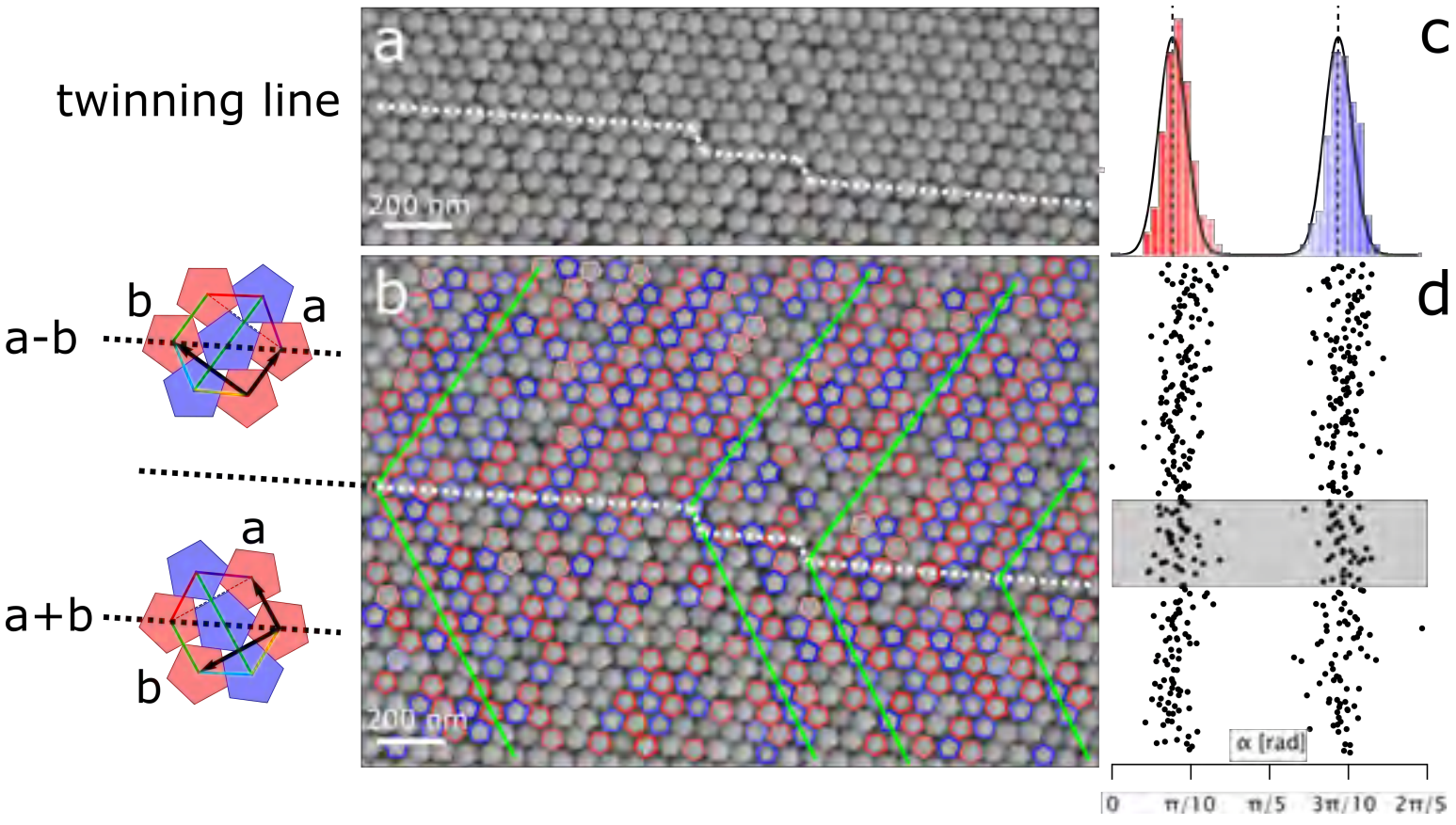
- The ice-ray and Dürer structures can coexist,
- with minimal lattice deformation!
- The (denser) ice-ray structure exhibits higher orientational order

Intermediate polymorphs



- Relative position shift of neighboring particles,
- while the orientation is preserved

Domain twinning



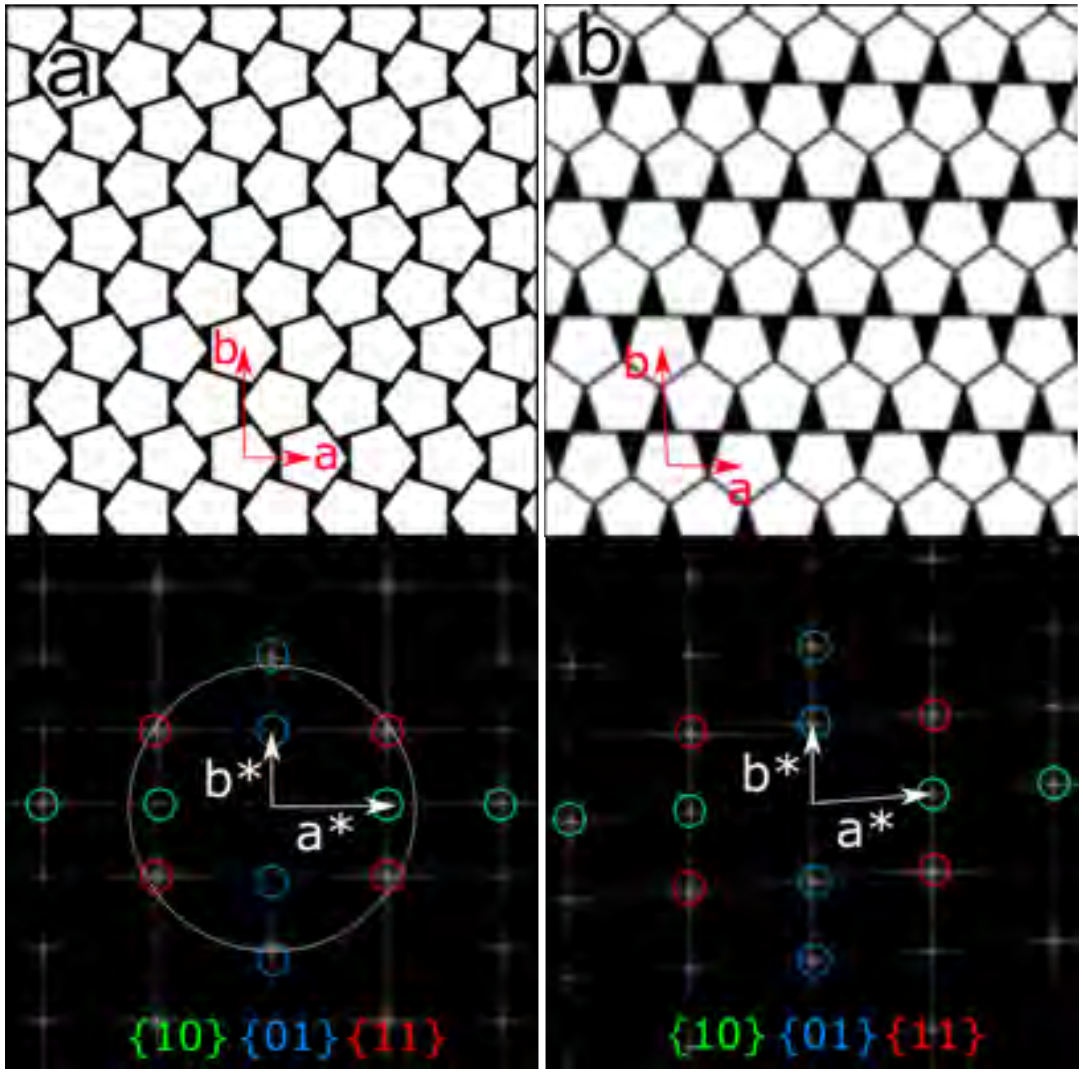
- The position is mirrored across the twinning line,
- while the orientation is preserved

Conclusion

- Clear evidence for ice-ray, Dürer and intermediate structures
- Subtle interplay between orientational and positional order
- Size effect (competition between overall compression and short-scale interaction ?)

J. Marcone et al., *Nano Letters* **23**, 1337-1342 (2023).
DOI: 10.1021/acs.nanolett.2c04541

Unit cell



Ice-ray (rectangular)

Dürer (oblique)

Particle size and disorder